Inverse problems and imaging

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Electrical Impedance Tomography (EIT)

EIT is a portable, inexpensive, non-invasive imaging system that requires no ionizing radiation.

- Attach electrodes to the **surface** of the domain.
- Apply (harmless) current and measure the resulting voltage.
- Recover the internal conductivity (and permittivity) by solving a PDE

$$\nabla \cdot \sigma(x) \nabla u(x) = 0, \quad x \in \Omega \subset \mathbb{R}^n$$

Applications include: breast cancer detection and tumor classification, monitoring lung function in ICU patients, stroke classification*, etc.

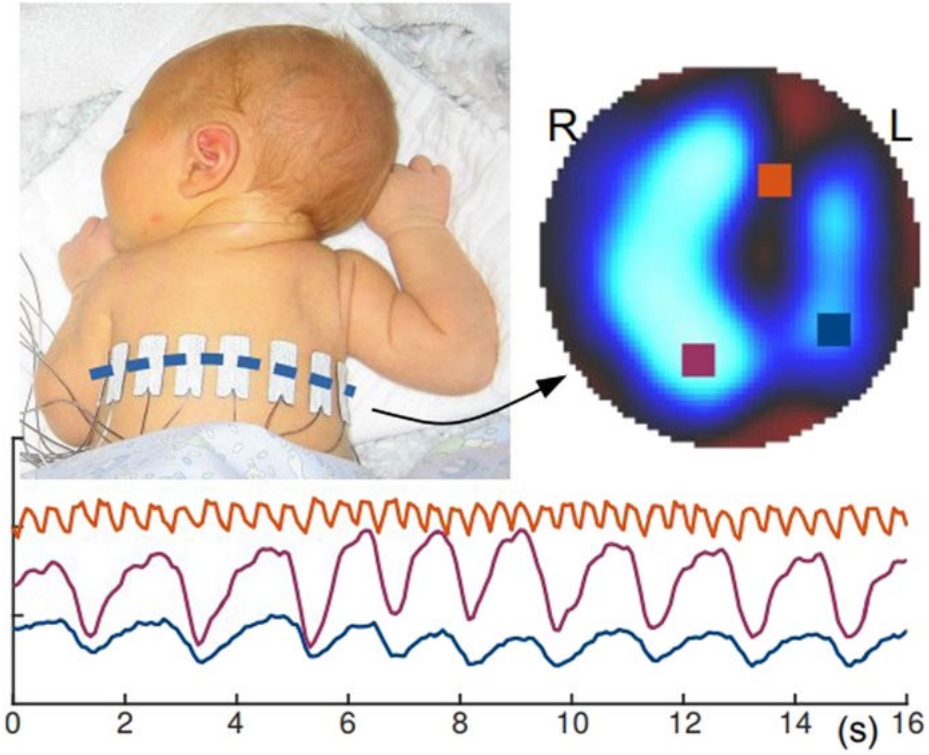
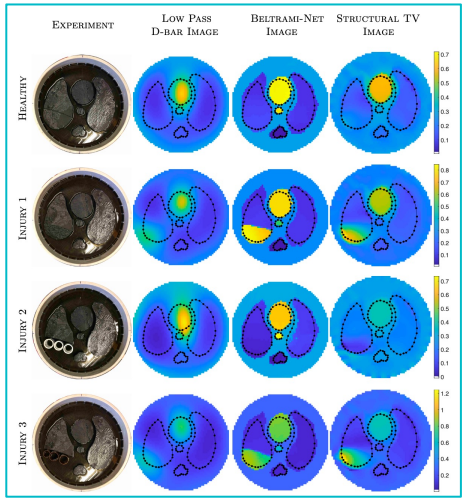


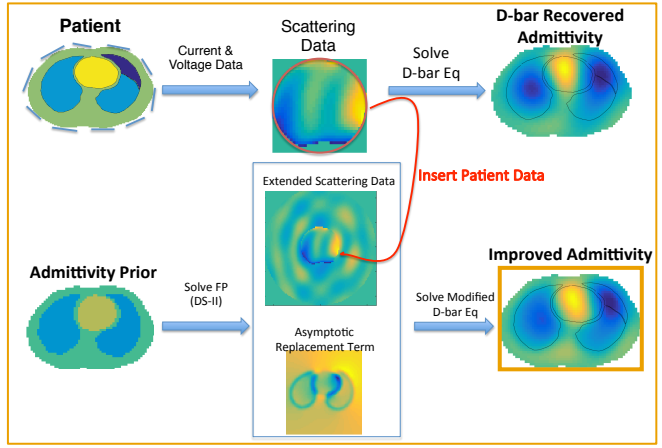
Figure: Heinrich, S., Schiffmann, H., Frerichs, A., Klockgether-Radke, A., and Frerichs, I. (2006). Body and Head Position Effects on Regional Lung Ventilation in Infants: an Electrical Impedance Tomography Study. *Intensive Care Med.* 32 (9), 1392–1398. doi:10.1007/s00134-006-0252-0

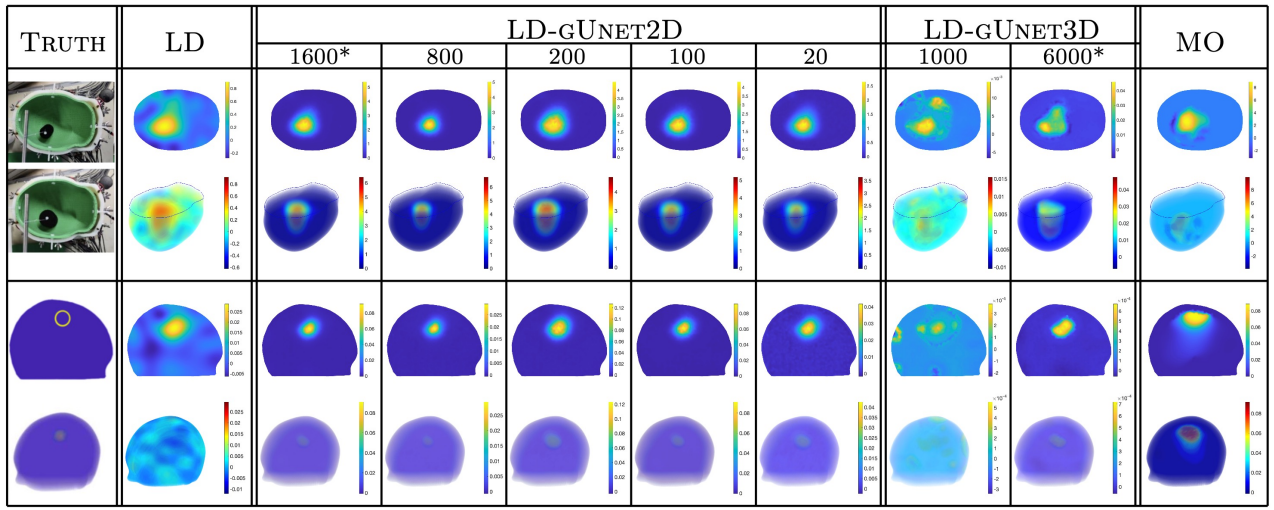
Research Focus

Development of fast algorithms that reliably recover sharp images from practical EIT data.

- Real-time image reconstruction
- Complex Geometrical Optics (CGOs)
- Including anatomical atlases
- 3D data (stroke and breast imaging)
- Deep learning (CNNs + GNNs)
  - Model-based learning
  - U-nets
  - Generalizability***







Hamilton et al, Beltrami-Net 2019

Preprint [H, Hauptmann, Kolehmainen, Toivanen 2025]

Helpful Skills

Math: PDEs, linear algebra, numerical analysis, inverse scattering theory (nonlinear Fourier transforms), regularization methods, functional analysis.

Prob/Stat: Deep learning, CNNs, Bayesian inversion.

Computing: HPC, GPU computing, programming with python, Matlab, C, etc.

An **inquisitive** mind + can-do attitude.

Further Reading (with links)

Foundational Work in EIT: Isaacson et al. 2004, IEEE TMI

Network Papers: Deep D-bar, Beltrami-Net, GCNM Model-based Learning, Graph U-nets

A priori D-bar: Full boundary, partial boundary.

3D CGO Papers: Sims 2020, Exper 2022

Linear and Nonlinear Inverse Problems with Practical Applications by Mueller & Siltanen

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